

Credit Channel of Monetary Transmission

The Role of Industrial Interenterprise Arrears

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NON-TECHNICAL SUMMARY

Many economic processes observed in Russia are similar to those in the transitional economies of Eastern Europe. However, the depth and length of Russia's output decline, together with its persistently high inflation have proved to be a challenge to explain

The existing literature on the output collapse (see, *e.g.*, Bruno (1992), Kornai (1994), Calvo and Kumar (1994), Granville (1995), Blanchard (1996), Sachs (1996b)) offers at least three general causes.

One set of explanations is associated with the shock of systemic transformation and the legacy of planning (*e.g.*, the change from supply-determined to demand-determined markets; the deep changes in the structure of relative prices following liberalization; and the disruption of coordination mechanisms during the movement away from central planning toward markets).

A second group of explanations focus on the influence of two macro shocks: the external trade shock from the dismantling of the Council for Mutual Economic Assistance (CMEA) and the dissolution of the USSR, which diminished both demand for products and input availability; and the post-Cold-War shock, which changed the structure of domestic demand following demilitarization.

A third set of explanations may be called policy-induced causes, since they are associated more with specific policy characteristics than those in the first two groups. One area of focus is trade liberalization, which increased international competition. Another is the sharp reduction in governmental subsidies (explicit and implicit), as well as centralized investment and defense spending. And a third is the monetary contraction.

Recognizing the significance of all the three groups of factors, we focus on the possible channels through which monetary policy contributed to the output decline in Russia. We believe that the decline related to the monetary contraction may be significant, given the underdevelopment of the credit market.

It is hardly possible to judge whether or not monetary policy can impose real costs on a transitional economy without proper understanding of transmission mechanisms. The underdevelopment of the financial sector alters the propagation mechanisms of monetary impulses. Given the underdevelopment of capital markets in Russia, the lending channel of

monetary policy transmission (modified in the context of a transitional economy) appears to be dominant at the initial stages of transition, whereas the interest rate and (traditional) exchange rate channels are likely to become significant only after the government Treasury bills' (GKO) market has developed; the so-called other asset price and balance sheet effects are hardly observable at all.

The lending channel, which is likely to remain the main channel in the initial stages of transition, is significantly modified in comparison with the one in developed economies, since credit extenders and recipients are quite different. On the supply side, the lack of adequate substitutes for loans from the CBR intensifies the decline in the supply of credit during periods of monetary tightening. On the demand side the lack of substitutes for bank loans other than trade credit deepens the enterprise side reaction.

Interenterprise arrears, being a "cushion" for enterprises in periods of tight liquidity, but not a perfect substitute for bank credit, complicate the monetary impulse propagation. We examine the role of interenterprise arrears using a two-sector (energy vs manufacturing) non-Walrasian general equilibrium model with price regulation and credit rationing.

According to the model, when the credit limit to a sector is relaxed, the sector's payment ratio increases, thus facilitating the other sector's production. The resulting steady-state inflation is not symmetric across the sectors: it increases if the first (energy) sector receives additional credit, and decreases if the credit is extended to the consumption good producer. If compared with the model without arrears, it turns out that interenterprise arrears may soften or strengthen the production-side reaction in the steady state, depending on respective elasticities. The latter is likely to depend on the characteristics of the production function and the transaction cost function, and the level of real money balances.

The model allows one to explain the sectoral pattern of decline, as well as the observed first three¹ waves of arrears. The model suggests that the depth of the recession in Russian industry, and its sectoral pattern, may partially be explained by the policy of excessive credit rationing, especially of the manufacturing sector. The model highlights the mechanisms through which policy, meant to fight inflation, could cause an increase in inflation together with a decline in both sectors' output and consumption.

¹ The first wave followed the initial money balances reduction, the second appeared after real interest rates became positive, and the third one after the government increased its borrowing on GKO market.

The model suggests that a relaxation of the credit rationing constraint to the second sector, and/or a decrease in interest rates, may be growth promoting strategies, and the former may even not result in higher inflation, implying that there is a rationale for governmental intervention.

Overall, it is likely that monetary policy, along with other factors, has added to the depth of the observed output decline in Russia from 1992 to 96. The sectoral pattern of the recession seems to have been influenced by the monetary impulse as well. The role of the CBR during the first years of transformation, when the new bank system was weak and unable to create credit resources on a scale required by an economy with a high share of industrial production, continues to be dominant. The transfer of responsibilities to the weak financial system resulted in a lack of liquid financial assets in the system, which was later translated into an extremely high interest rate on money resources and the outburst of quasi-money.

1. INTRODUCTION

Many economic processes observed in Russia have been similar to those in the transitional economies of Eastern Europe. However, the reasons for the depth and the length of its transformational recession, together with its persistently high inflation, have been difficult for economists to pin down.

There are a variety of reasons for output decline that have been recognized in the transitional economics literature (see, *e.g.*, Bruno (1992), Kornai (1994), Calvo and Kumar (1994), Polterovich (1995a), Blanchard (1996), Sachs (1996), Gomulka (1998)). Recognizing the significance of many other factors, we will focus on the possible channels through which monetary policy may have influence the output decline in Russia. It is likely that the portion of the output collapse related to the monetary contraction may be significant.

There is no agreement in the academic literature about the relationship between the tightening of monetary policy from 1992 – 1996, the output decline, and the role of interenterprise arrears. Some authors believe that widespread interenterprise credit is a normal substitute for banking credit, and there is no special "arrears' problem" in the Russian economy since the ratio of trade credit to GDP is not abnormal by international standards (Alfandari and Schaffer (1996)). However, this view can be challenged on a number of grounds.

First, it is difficult to evaluate what is a normal level of trade credit, and particularly, overdue trade credit. In an economy in transition a large part of the stock reflects uncollectible receivables from firms that will never pay. Moreover, the tradable value of a Russian receivable is much lower than in an economy with a developed capital market.

Second, it seems incorrect to discuss the level of trade credit in an economy in isolation from other indicators, the level of bank credit in particular. In Russia, the ratio of total trade credit to GDP is twice the level of total banking credit. Total trade credit (receivables and payables) in Russia in 1994 – 1997 was 30 – 40% of annualized GDP (47% in 1996). Overdue trade credit amounted to about 20%, not an exceptional amount relative to international standards. Total domestic credit to the private sector was only about 10 – 15% of GDP during the same period. By comparison, the ratio of total trade credit to bank credit to

the private sector was 0.5 in Canada and the USA and 0.33 in the UK², suggesting that it is hardly possible to evaluate the level of trade credit in Russia as normal. The level of trade credit in Russia is comparable to that in Poland, which is believed to have experienced output losses following credit contraction (Calvo and Coricelli (1993)).

Moreover, with internal finance being dominant, bank loans are important in corporate finance for developed economies, with their share in gross financing varying from 13% in Canada to more than 40% in France and Japan. In most developed countries, trade credit is only the fourth most important source of finance, after internal resources, bank loans and securities. Exceptions include Finland and Japan where securities are relatively less important (see Calvo and Kumar (1993): Appendix Table 1, p. 30; Mayer (1991): Table 12.3, p. 312).

The financing pattern of Russian industrial enterprises differs sharply. Interenterprise trade credit is the dominant source of short-term finance of industrial enterprises. Payables to suppliers accounted for almost 50% of total industrial enterprises' liabilities in 1993 – 1995, with about half being overdue, whereas banking credit and internal financial resources (real money balances and securities holdings) accounted for roughly 20% and 15% of total liabilities, respectively (Table A1 in Appendix).

It is almost impossible to evaluate whether monetary policy has real costs without properly understanding the transmission mechanisms in a transitional economy. The general underdevelopment of financial markets in transitional economies, and money and credit markets in particular, together with widespread interenterprise arrears, are likely to distort the monetary transmission mechanisms and, if not taken into account, make monetary policy unreliable and less effective.

The objective of this paper is to clarify the role of interenterprise arrears in the propagation mechanism. Additionally, we will try to understand whether monetary issues, and the credit regime in particular, could have affected the sectoral pattern of the industrial recession.

It is generally accepted that monetary policy can have a significant influence on real economic variables in the short run, although the propagation mechanisms for any monetary impulse are debated. There are four major types of monetary channels in a developed economy recognized

² Author's calculations based on *International Financial Statistics* (1997: pp 572–575) for Russia and *International Financial Statistics* (1996: pp 166, 222, 254, 506, 638, 642) for the rest of the countries. Trade credit data is from Alfandari and Schaffer (1996): pp 104–105.

in literature (see, e.g., Mishkin (1995a) for an overview): the interest rate channel, originally associated with the basic Keynesian IS-LM model; the exchange rate channel, operating through net exports; the so called "other asset price effects"; and the credit channel. The latter stresses the influence of imperfect information in financial markets and, in particular, the fact that informational asymmetries result in equilibrium credit rationing (Kashyap *et al.* (1993), Bernanke and Blinder (1992)).

Denisova (1997) shows that given the under-development of the stock market in Russia, the lending channel of monetary policy transmission appears to have been dominant during the initial stages of transition, whereas the interest rate and exchange rate channels are likely to have become significant only after 1995 and the development of the market for government Treasury bills (GKO and OFZ); the other asset price and balance sheet effects are hardly observable at all.

The lending channel of monetary policy transmission appears to have been intensified by Russia's weak private banking system and the absence of substitutes for the CBR's loanable funds, on the supply side³, and the lack of substitutes for bank loans, other than trade credit, on the demand side.

Can arrears provide an adequate substitute for banking credit? As shown in the literature on transitional economics, there are several features that distinguish interenterprise arrears from trade credit in a market economy. First, there is a large involuntary component of holding arrears, since there is practically no possibility to recover them legally. Second, interest rates on arrears are very low. Third, the degree of mutual indebtedness is very high.⁴

Since arrears are practically interest-free, may be they are even a better source of short-term finance than banking credit? What is the effect the institution of arrears on growth and inflation in the context of a transitional economy characterized by the dominance of short-term interests, the continued regulation of certain prices (e.g., on energy products in

³ Relative to those in developed countries, the assets of Russian banks are characterized by a high share of reserves and claims on the government. A lack of securities, together with lower shares of deposits and money market instruments, are the major differences on the liability side. Relative to banks in other transitional economies, the main distinction is the lower share of centralized resources (*i.e.*, credit from monetary authorities and government deposits) (Denisova (1997): Table 4, p. 38–39).

⁴ It is not that large firms extend trade credit to credit rationed small firms like it happens in market economies. It is rather a complex network of mutual arrears.

Russia), and the separation between household and enterprise money? To clarify the role of arrears in monetary transmission, an analytical framework is needed.

There are few studies that deal with the problem of interenterprise arrears itself, and fewer attempts at formally modeling how the mechanism of arrears affects transmission (Blinder (1987), Granville *et al.* (1996), Kim and Kwan (1995)).

The model suggested below may be considered a counterpart of the one in (Granville *et al.* (1996)), where the credit regime under which the interest rate plays the allocative role was studied. The model is simplified to exclude competition from an imported good. The framework provides an opportunity to trace the influence of different policies on real production and inflation in the complex environment of a transitional economy. In particular, by emphasizing the transactional role of liquid assets, the framework allows one to consider the real effects of monetary tightening and the shift to quasi-money (arrears). Distinguishing between two sectors — an energy sector and a manufacturing sector — helps one to clarify those factors that determine the dynamics of the sectoral pattern of output.

The introduction of credit rationing into the model changes several relationships between variables and, according to the results of comparative statics' analysis, allows one to draw stronger conclusions about the influence of credit tightening on output and inflation than were possible in (Granville *et al.* (1996)). In addition, the existence of two monetary circuits⁵ is taken into account.

⁵ Enterprises still face strict regulations on converting non-cash accounts into cash. This is a legacy of the planning era.

2. A TWO-SECTOR MODEL

2.1. The structure of the model

We consider five economic sectors in the economy: an energy sector, a manufacturing sector, a banking sector, a household sector and a government sector. There are two domestically produced goods. The economy's real flows are presented in Fig. 1. The exhaustive list of variables used in the model is presented in Appendix B.

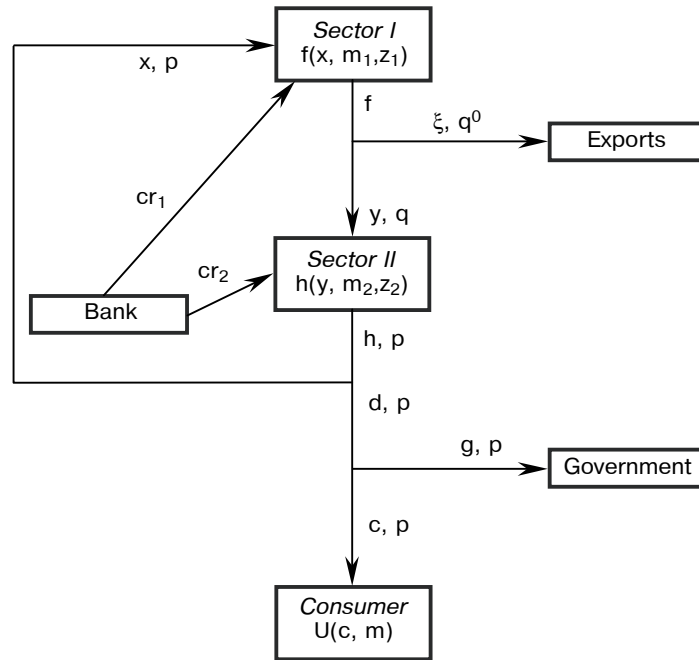


Figure 1. Real flows in the modeled economy.

Exogenous variables: $q, R, \varepsilon, q^0, g, \theta, cr_1, cr_2, M_0$.

Endogenous variables: $x, m_1, z_1, f, y, m_2, z_2, h, c, m, \xi, d, p$.

The first sector produces energy resources (f), a portion of which are used as working capital by the second sector (y). The remainder is ex-

ported⁶ (ξ). The second sector produces the manufactured good (h), which is demanded as working capital by the first sector (x) and as a consumption good by households (c) and the government (g).

The price of energy, q , is regulated and is exogenously fixed for a period.⁷ The price of the manufactured good, p , is determined by supply and demand.

The banking sector decides upon the interest rate (R) and credit limits to the first sector (Cr_1) and to the second sector (Cr_2). The bank sector's behavior is not modeled explicitly.⁸ We assume these values to be exogenously given.

Working capital is assumed to be the only physical input in the production process. The transactional role of money is taken into account by introducing real money balances into the production function. It is assumed that money balances can be increased only from the external source of banking credit.

To reflect the widespread interenterprise arrears we will use the production function modification⁹ proposed by Victor Polterovich in (Granville *et al.* (1996)). In terms of the first sector variables, the production function is of the form: $f(x, m_1, z_1) = \Omega(\bar{x}) = \Omega(x - \mu(z_1 / m_1) z_1)$,

⁶ It is assumed that the world price of energy resources is higher than the domestic one: $\varepsilon q^0 > q$. Therefore, exports would be attractive for producers but they are subject to quotas, and the restriction is binding. Consequently, the first sector does not choose the amount of ξ , which is exogenously given to the producer. The export quota ξ is determined residually in the model: given the exogenously determined price q , it is the difference between production f and domestic demand y , *i.e.* $\xi = f - y$.

⁷ Exports, in the form in which they are introduced, offer a way to escape difficulties of modeling imbalances in the market for the first good, given that the price on the good is fixed. Exports can be viewed as a premium available for producers after satisfying domestic demand. In this sense, the economy is effectively a closed one.

⁸ There is no intertemporal optimization in the model, and thereby no explicit motive for savings. This restricts the role of the banking system in the model, but — to repeat — the actual size of bank credits is extremely low in the private sector. Russian banks are much more active on the market for government bonds, which we do not want to consider in the model.

⁹ A more standard production function, such as Cobb–Douglas, is too "rigid" to capture some important empirically-supported relationships (*e.g.*, the dependence of the optimal payment ratio (defined below) on the level of the money balances available to producer).

where x is working capital, m_1 represents real money balances, z_1 is working capital paid for, μ is a transaction cost function, and \bar{x} is actual (*i.e.* adjusted for transaction costs) working capital.

The key point is that there are transaction costs of buying a unit of working capital, which are decreasing with higher levels of real money balances. Costs increase in the amount of the input bought, which makes payment arrears attractive. Actual working capital used in production (\bar{x}) is less than the working capital demanded by the amount of transaction costs.

Assume further that there is a penalty for having arrears. The penalty for a unit of arrears decreases when the payment ratio, defined as $\gamma = z_1/x$, increases. The penalty function ϕ is assumed to approach infinity when payments are zero. When real money balances fall below a certain critical level it becomes profitable to have payment arrears even if there is a penalty for holding overdue trade credit.¹⁰

It is possible to think about the suggested setting in terms of the transitional economy's analogue of the external finance premium. In periods of credit tightening, and therefore a squeeze on the real money balances of enterprises, the wedge between the cost of running trade credit arrears and borrowing on the credit market increases in favor of trade credit arrears.

Informational aspects of the interenterprise arrears problem in a transitional economy are beyond our consideration, and complete information is assumed.

2.2. Short-run equilibrium

Producers decide upon actual inputs \bar{x} and \bar{y} , and thus outputs f and h , for one period. In the presence of transaction costs and the possibility of not paying for the input in full, producers choose both the amount of working capital (x) they want to get from the other sector and the amount of working capital they intend to pay for (z_1), together with the level of real money balances they want to have to facilitate the purchase of the input (m_1). Since the money balances can be increased only by borrowing from the bank, producers need to pay interest on the credit they obtain. Producers must also pay a penalty for having arrears, which is transferred to the other sector. Hence, the first (energy) sector's ob-

¹⁰ The condition for the payment ratio to be strictly less than 1 is derived in the model.

jective function can be written in the following way (where the last term is the penalty payment from the second sector transferred to the first sector):

$$\begin{aligned}
 \max_{x, \Delta m_1, z_1} \Pi(x, \Delta m_1, z_1) &= \\
 &= \max_{x, \Delta m_1, z_1} \varepsilon q^0 \xi + \beta q[f(x, m_1, z_1) - \xi] - \\
 &\quad - p\phi(x/z_1)(x - z_1) - pz_1 - Rp\Delta m_1 + q\phi(y/z_2)(y - z_{21}) = \\
 &= \max_{x, \Delta m_1, z_1} \varepsilon q^0 \xi + \beta q[f(x, m_1, z_1) - \xi] - \\
 &\quad - p\phi(x/z_1)(x - z_1) - pz_1 - Rp\Delta m_1 + q\phi(1/\beta)(1 - \beta)y, \quad (1) \\
 \text{s.t. } \Delta m_1 &\leq \frac{Cr_1}{p}, \quad m_1 = m_{10} + \Delta m_1,
 \end{aligned}$$

where β is the second sector payment ratio, *i.e.* the ratio of paid for to total demanded input $\beta = z_2/y$. Given that ξ and β are exogenous for the first sector and assuming that $\partial \beta^* / \partial \gamma = 0$ ($\gamma = z_1/x$), the first sector's objective function is equivalent to:

$$\max_{x, \Delta m_1, z_1} \beta q f(x, m_1, z_1) - p\phi(x/z_1)(x - z_1) - pz_1 - Rp\Delta m_1. \quad (1')$$

The second (manufacturing) sector's problem is symmetric to the first sector's if we assume $h(y, m_2, z_2) = \Omega(\bar{y}) = \Omega(y - \mu(z_2/m_2)z_2)$. The only difference is that the second sector's product is demanded not only by the first sector as working capital, but also as a consumption good by households and the government. The existence of payment arrears from the first sector is equivalent to a price reduction on the second sector's good (in comparison with the price charged to households and the government). For the sake of simplicity¹¹, let us assume that price arbitrage will result in equalizing the two prices ($p_c = \gamma p$)¹² so that we can write

¹¹ This assumption allows us to avoid introducing an extra variable that would reflect the second sector's choice of the fraction of its output sold to each of the two groups of customers.

¹² The presence of government arrears, reflecting the bargaining power of the government, allows it to drive the price for a unit of the second good to the lower bound (*i.e.*, the result of arbitrage is not that $\gamma = 1$, but that $p_c < p$).

the manufacturing sector problem as:

$$\begin{aligned}
 \max_{y, \Delta m_2, z_2} \Pi(y, \Delta m_2, z_2) = \\
 = \max_{y, \Delta m_2, z_2} \gamma p h(y, m_2, z_2) - q \varphi(y / z_2)(y - z_2) - q z_2 - \\
 - R q \Delta m_2 + p \varphi(x / z_1)(x - z_1), \\
 \text{s.t. } \Delta m_2 \leq C r_2 / q, \quad m_2 = m_{20} + \Delta m_2,
 \end{aligned} \tag{2}$$

where γ is the second sector payment ratio, *i.e.* $\gamma = z_1/x$. Given that γ is exogenous for the second sector and assuming that $\partial \gamma^* / \partial \beta = 0$ ($\beta = z_2/y$), the second sector's objective function is equivalent to:

$$\max_{x, \Delta m_1, z_1} \gamma p h(y, m_2, z_2) - q \varphi(y / z_2)(y - z_2) - q z_2 - R q \Delta m_2. \tag{2'}$$

The household sector decides upon demands for the consumption good and real money balances so as to maximize utility. The introduction of real money balances in the utility function is meant to capture the transactional role of money. A consumer's income comes from two sources: net of taxes profits¹³ and money from the previous period. Savings are made in the form of holding money balances, which can be thought of as interest-free accounts at a bank.¹⁴

The consumer's problem can be written as:

$$\begin{aligned}
 \max_{c, m} U(c, m), \\
 \text{s.t. } p c + M = I, \\
 I = (1 - \theta) \gamma p d + (1 - \theta) \varepsilon q^0 \xi + M_0, \\
 m = M/p.
 \end{aligned} \tag{3}$$

There are three balance conditions in the economy, which reflect the flows of physical goods:

$$f = \xi + y, \quad h = d + x, \quad d = g + c. \tag{4}$$

¹³ What is called profit in the model is actually value-added, since we do not separate between wages and profits.

¹⁴ This assumption is consistent with the fact that in 1992 and during the beginning of 1993 interest rates on private savings in Sberbank were far below inflation. However, we do not consider substitution into hard currency and dollar denominated accounts, which were common practices at the time.

It is worth noting that government expenditures g are financed via revenues from the profit tax, seigniorage and the gain in the price due to the existence of payment arrears¹⁵:

$$\begin{aligned} pg &= p \left\{ \theta \left[\gamma d + \frac{\varepsilon q^0}{p} \xi \right] - \frac{\varepsilon q^0}{p} \xi + \Delta m + (1 - \gamma) d \right\} = \\ &= p \left\{ (1 - \gamma(1 - \theta)) d + (\theta - 1) \frac{\varepsilon q^0}{p} \xi + \Delta m \right\}, \end{aligned} \quad (5)$$

where $\Delta m = (M - M_0)/p$.

The money market equilibrium is determined by the following conditions¹⁶:

$$\begin{aligned} \Delta M^s &= \Delta M^d + \Delta M_1 + \Delta M_2, \\ \Delta M^d &= M - M_0, \\ \Delta M_1 &\leq Cr_1, \\ \Delta M_2 &\leq Cr_2. \end{aligned} \quad (6)$$

The short-run equilibrium trajectory is a sequence of static equilibria in periods $t = 1, 2, \dots$

A static equilibrium is the set $\{x^*, z_1^*, m_1^*, f^*, \xi^*, y^*, z_2^*, m_2^*, h^*, \beta^*, \gamma^*, c^*, m^*, p^*\}$ such that:

- 1) x^*, z_1^*, m_1^*, f^* are the first sector value-added maximizing choices, given prices p^* and q , the nominal interest rate R , the credit limit Cr_1 and the second sector payment ratio β^* ;

¹⁵ (5) is easy to check using the consumer's budget constraint and the balance condition $d = g + c$.

¹⁶ Since banking system behavior is not modeled explicitly, this setting (in the case of binding credit limits) is equivalent to putting limits on the growth of the money supply. The latter would mean that supply of money is exogenous, while the sum of credit limits to the sectors is determined residually: $Cr_1 + Cr_2 = \Delta M^s - \Delta M^d$. There is no difference between the two schemes in the case of binding credit limits. When credit limits are not binding, it is assumed that the interest rate (which is exogenous in our framework) is such that money supply is equal to money demand from consumers and the banking system (the latter being equal to demand from producers).

- 2) y^* , z_2^* , m_2^* , h^* are the second sector value-added maximizing choices, given prices p^* and q , the nominal interest rate R , the credit limit Cr_2 , and the first sector payment ratio γ^* ;
- 3) β^* and γ^* constitute a Nash equilibrium with a Cournot-type conjectural variation assumption, *i.e.* γ^* is the first sector optimal payment ratio given the second sector optimal choice β^* and assuming $\partial\beta^*/\partial\gamma = 0$; and β^* is the second sector optimal payment ratio given the first sector optimal choice γ^* and assuming $\partial\gamma^*/\partial\beta = 0$;
- 4) c^* , m^* are the consumer's utility maximizing choice, given prices p^* , q , total profits Π^* , the tax rate θ and initial money balances M_0 ;
- 5) $(p/q)^*$ is the second product market clearing price ratio, *i.e.* such that $g + c^*((p/q)^*) = h^*((p/q)^*) - x^*((p/q)^*)$, where q is exogenously given;
- 6) ξ^* is export quota in equilibrium: $\xi^* = f^* - y^*$.

Given the assumptions made about the functions f , h , ϕ , μ and U , and assuming that the substitution effect caused by the price ratio change is stronger than the income effect, an equilibrium exists and is unique.

2.3. The model with symmetric credit rationing

Two credit regimes can be identified in Russia. In 1992 – 1993 the real interest rate was substantially negative, and credit was rationed. Since 1994 the real interest rate has become positive, and comprehensive credit rationing has stopped. However, there are indications of selective credit rationing aimed at overcoming adverse selection and moral hazard problems.

From this perspective it is likely that industrial sectors, which are substantially non-symmetric in their financial positions (*e.g.*, the share of the fuel sector in total industrial profits was about 20 – 25% in 1992 – 1994 (Belousov and Klepach (1995): Table 1, p. 55)), may be treated differently by banks. Banks are typically more reluctant to lend to manufacturing sector enterprises, which are much less competitive on the world market than energy sector companies. The fact that the share of overdue bank credit (both as a percentage of GDP, and as a percentage of total credit extended to the sector) is higher in the manufacturing sector than in the energy sector (Table A1 in Appendix), may also suggest that banks would try to monitor firms in the manufacturing sector, and thereby practice credit rationing. Firms in the energy sector are less likely to experience additional restrictions.

The empirical analysis of industrial data for 1994 – 1995 presented in Denisova (1997) indicates that there is a substantial asymmetry in financing patterns between industrial sectors: the energy sector behavior regarding bank credit is governed by the real interest rate, whereas the manufacturing sector appears to be credit rationed. Overdue trade credit is likely to exert little influence on the energy sector's output dynamics, while it is important for the manufacturing sector. It seems reasonable to take into account the observed asymmetry between the sectors in the model, that would allow one to analyze the influence of different patterns of short-term finance on the variance in output decline across the sectors.

To reflect the two credit regimes, we will analyze two versions of the model: both sectors are credit rationed (the first version); the energy sector is not credit rationed, while the manufacturing sector is rationed¹⁷ (the second version). Formally, depending on different credit regimes, we will allow:

- $\Delta m_i = cr_i$, *i.e.* the credit constraint is binding for producer i and there is credit rationing;
- $\Delta m_i < cr_i$, *i.e.* the constraint is not binding for producer i and there is no credit rationing.

The model with symmetric credit rationing is analyzed in Section 2.3.1 (short-run version) and Section 2.4.1 (long-run version), while the model with asymmetric credit rationing is presented in Section 2.4.2 (long-run version).

To simplify the analysis, we will assume in what follows that the production function, the transaction cost function and the penalty function have the following forms¹⁸:

$$\Omega(\lambda) = b\lambda^\alpha, \mu(\lambda) = a\lambda^\nu, \varphi(\lambda) = k\lambda^\sigma, \alpha \in (0,1), \sigma \in (0,1).$$

2.3.1. Short-run comparative statics. With the above parametrization and under the credit rationing regime, the first sector (energy pro-

¹⁷ Here we assume that the microfoundation for credit rationing of one of the sectors is that credit rationing of enterprises (as an optimal strategy for banks given the informational asymmetries they face (Stiglitz and Weiss (1981)) occurs more frequently in the manufacturing sector than in the energy sector.

¹⁸ The constraints on α and σ guarantee that the objective function is strictly concave in terms of the payment ratio γ , and thus first order conditions give a maximum of the objective function, and the optimum is unique.

ducer) problem can be stated as:

$$\begin{aligned} \max_{x, z_1} & \beta q b(x - a z_1^{v+1} m_1^{-v})^\alpha - p k(x^{1+\sigma} z_1^{-\sigma} - x^\sigma z_1^{1-\sigma}) - p z_1 - R p c r_1, \\ \text{s.t. } & \Delta m_1 = c r_1; m_1 = m_{10} + \Delta m_1. \end{aligned} \quad (7)$$

The second sector problem is symmetric. The first-order conditions for producers' problems are derived in Appendix C. It is worth noting that the FOCs are derived in terms of γ and β (the endogenously determined ratio of sector input paid for). Under certain constraints on parameters, the optimal payment ratios are strictly less than 1. That is, $\gamma^* < 1$ and $\beta^* < 1$, and it is optimal for the sectors not to pay for their inputs in full.

To study the influence of the exogenous variables on the short-term equilibrium, it is convenient to define the supply and demand for the second commodity in the following way:

$$S = d = h(p/q, \beta, \gamma, m_2, k) - x(p/q, \beta, \gamma, m_1, k) \text{ and } D = d = g + c(p/q, l/q).$$

In addition, we can define the export function

$$\xi = f(p/q, \beta, \gamma, m_1, k) - y(p/q, \beta, \gamma, m_2, k).$$

The signs of the derivatives of the supply function and the export function are easily derived, using results on the signs of partial derivatives presented in Table C1 in the Appendix, and the signs of the derivatives of the demand function are discussed in Appendix E5.

$$\begin{aligned} S &= S(p/q, \beta, \gamma, m_1, m_2, k); & \xi &= \xi(p/q, \beta, \gamma, m_1, m_2, k); \\ &+ \quad ? \quad ? \quad - \quad + \quad ? & - \quad ? \quad ? \quad + \quad - \quad ? \\ D &= D(g, p/q, \varepsilon/q, \theta, M_0/q, \gamma, \beta, k, c r_1, c r_2). \\ &+ \quad - \quad + \quad - \quad + \quad ? \quad ? \quad ? \quad + \quad - \end{aligned} \quad (8)$$

The supply of the second good is an increasing function of the price ratio (p/q) and the second sector credit limit, and a decreasing function of the first sector credit limitation ($c r_1$). The effects of the payment ratios (γ and β) and the penalty function coefficient (k) are ambiguous. The demand for the domestic consumption good depends positively on government expenditures in real terms (g), the real exchange rate (ε/q), the first sector credit limit ($c r_1$) and previous period real money balances¹⁹

¹⁹ This is a result of price rigidity.

(M_0/q). Demand is a negative function of the domestic price ratio (p/q), the profit tax rate (θ) and the credit limit to the second sector in real terms (cr_2). The influence of the payment ratios (γ and β) and the penalty function coefficient (k) are ambiguous.

Given that the static equilibrium is unique, we can trace out the effects of changing exogenous parameters. The results are summarized in Table 1.

Table 1. Sign changes of main variables in response to parameters' variation (short-run comparative statics results).

Variable Parameter	p/q	x	z_1	f	ξ	y	z_2	h	c
g	+	-	-	-	-	+	+	+	-
cr_1	+	?↑	?↑	?↑		+	+	+	-
cr_2	-	+	+	+		?↑	?↑	?↑	+
θ	-	+	+	+	+	-	-	-	-
k									

?↑ means that the influence is likely to be positive, provided that the effect of relaxing the credit limit is stronger than the relative price effect.

An increase in g . An expansion of government expenditures shifts the demand curve outwards (Fig. 2), with the supply curve unchanged. The equilibrium price ratio $(p/q)^*$ and d^* increase. The price ratio increase raises the optimal first sector payment ratio γ and diminishes the second sector ratio β . The change in the price ratio and the diminished payment ratio of the second sector β result in a decrease of first sector output (f) and of exports (ξ), while the second sector output (h) increases both as a result of both the price ratio change and the first sector payment ratio (γ) enlargement. Consumption c is likely to decrease (if the substitution effect outweighs the income effect).

An increase in θ . An increase in the tax rate shifts the demand curve inwards, with the supply curve unchanged. The resulting equilibrium price ratio $(p/q)^*$ and d^* are lower. The fall of the price ratio diminishes the first sector payment ratio γ and increases the second sector ratio β . The change in the price ratio and the increased payment ratio of the

second sector β cause the first sector output (f) and exports (ξ) to increase, while the second sector output (h) falls as a result of both the price ratio change and the first sector payment ratio (γ) decline. Since government expenditures (g) are not changed, while d^* diminishes, consumption of the domestic good (c) decreases. The effect on m is ambiguous because the change in income is ambiguous.

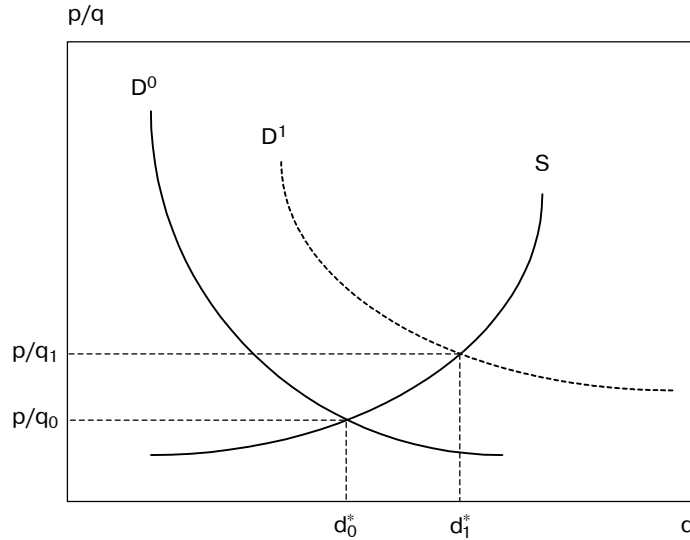


Figure 2. Short-run effects of increased government expenditures.

An increase in cr_1 . Relaxing the first sector credit limit shifts the supply curve leftwards. The demand curve shifts rightwards. The equilibrium price ratio $(p/q)^*$ increases, while the change in d^* is ambiguous. Given that the marginal propensity to consume out of income is less than one, one may conclude that the shift of the demand curve is of minor effect in comparison with that of the supply curve (Fig. 3). The price ratio increase raises the optimal first sector payment ratio γ and diminishes the second sector ratio β . The relaxation of the first sector credit limit adds to the increase of the payment ratio of the sector (γ).

The change in the price ratio and the diminished payment ratio of the second sector β result in a decrease of the first sector's output (f) and exports (ξ), while the increased real money balances of the first sector facilitates both production and exports (via the increased γ). The overall

effects on f and ξ are ambiguous.²⁰ The second sector's output (h) increases as a result of both the price ratio change and the first sector payment ratio (γ) enlargement. The second sector output (h) expands. As government expenditures (g) are not changed, while d^* diminishes, consumption of the domestic good (c) will decrease. The effect on m is ambiguous.

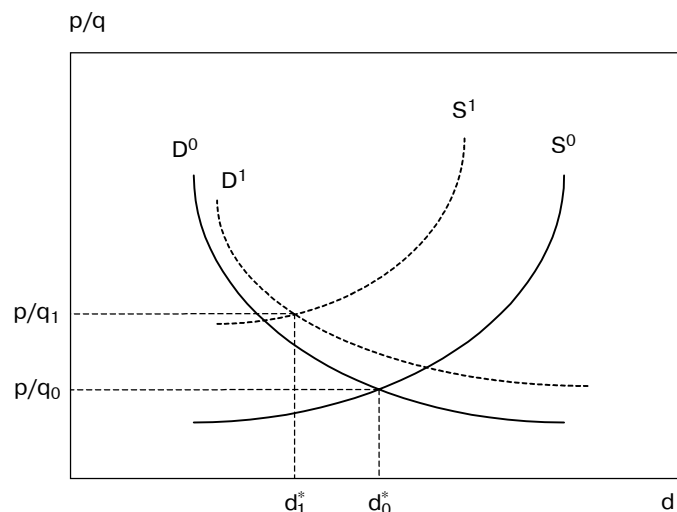


Figure 3. Short-run effects of relaxing the credit limit to the first sector.

An increase in cr_2 . A relaxation of the second sector's credit limit shifts the supply curve rightwards and the demand curve downwards. The same argument about the marginal propensity to consume out of income suggests that the shift in the demand curve is of minor effect compared to that of the supply curve (Fig. 4). The equilibrium price ratio $(p/q)^*$ decreases, while d^* increases. The fall in the price ratio causes the optimal first sector payment ratio γ to diminish and increases the second sector ratio β . The relaxation of the second sector's credit limit adds to the increase of the payment ratio of the sector (β).

²⁰ If the credit rationing constraint is too restrictive, one could expect the effect of relaxing the credit limits to outweigh the price ratio change. Indeed, if m_1 is small, $\partial f / \partial m_1$ is large. This could result in the expansion of the first sector's production. The effect on exports will still be ambiguous.

The change in the price ratio and the increased payment ratio of the second sector β cause the first sector output (f) and exports (ξ) to increase, while the second sector output (h) tends to diminish as a result of both the price ratio change and the first sector payment ratio (γ) decline. However, the increased real money balances of the second sector facilitates the production of the second good (via the increased β), and thus causes exports to decline. The overall effects on h and ξ are ambiguous.²¹ The first sector's output (f) expands. The unchanged government expenditures (g) allows consumption of the domestic good (c) to increase. The effect on m is ambiguous.

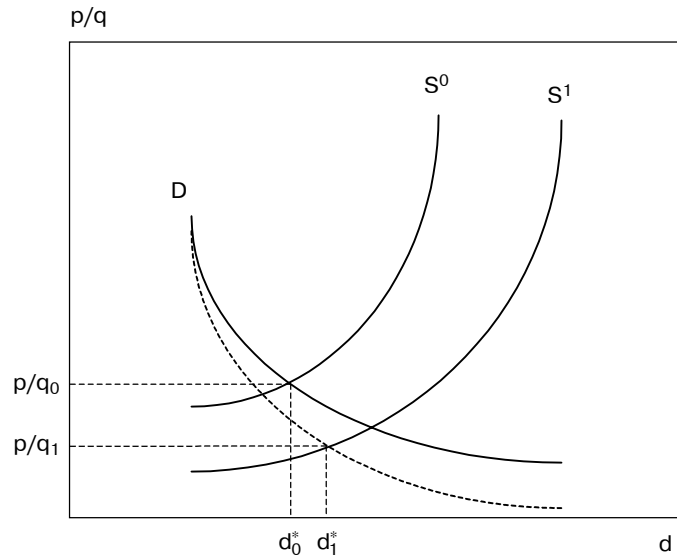


Figure 4. Short-run effects of relaxing the credit limit to the second sector.

An increase in k . The changes generated are ambiguous since the direction of the shifts of both the supply and demand curves are ambiguous.

²¹ If the credit rationing constraint is very restrictive, and therefore $\partial h / \partial m_2$ is large, the effect of relaxing the credit limits may outweigh the price ratio influence, and thereby cause the second sector's production to increase. The change in exports is ambiguous.

2.4. Long-run equilibrium

The dynamic version of the model is easily obtained from the static version if we define time paths of the exogenous variables, since it is assumed in the framework that the behavior of the economic agents is essentially short-term-determined. The latter reflects the fact that the high degree of economic and political uncertainty made them survival-oriented and replaced intertemporal considerations with short-term objectives.

Let us assume that tax rate θ and real credit limits to both sectors (cr_1 and cr_2) do not change over time²², while domestic (regulated) prices²³ are changed with reference to the inflation rate. We will consider two possibilities²⁴:

- the first sector price and nominal exchange rate follow the second sector price inflation with a one period lag, *i.e.*

$$q_t = (1 + \pi_{t-1})q_{t-1}, \quad \varepsilon_t = (1 + \pi_{t-1})\varepsilon_{t-1}. \quad (9)$$

- the first sector price and nominal exchange rate are changed in line with inflation in the second sector, *i.e.*

$$q_t = (1 + \pi_t)q_{t-1}, \quad \varepsilon_t = (1 + \pi_t)\varepsilon_{t-1}, \quad (9')$$

where $\pi_t = p_t / p_{t-1} - 1$.

The nominal interest rate is known to be set so as to keep the real interest rate constant: $1 + R_t = (1 + \rho)(1 + \pi_t)$. The real interest rate ρ is exogenously given. The indexation rules are known to all economic agents.

It is also assumed that credit is extended for one period. The interest is paid back in the next period and at the rate prevailing in period $t+1$. In this case, expectations are important. For simplicity we will assume perfect foresight: $\pi_{t+1}^e = \pi_{t+1}$.

²² This is equivalent to assuming that in the steady-state the nominal credit limit to a sector is growing with the rate of steady-state inflation.

²³ This includes the nominal exchange rate.

²⁴ It is shown in (Granville *et al.* (1996)) that these assumptions are consistent with empirical data.

The way arrears are introduced in the model has one major deficiency: it is assumed that after the end of period t the accumulated trade credit debt, equal to $(x_t - z_{1t})$ (in terms of the first sector) is not transferred to the next period, and in this sense is never repaid. Therefore, it is rather non-payment, and not trade credit or late payment. However, the assumption that the penalty for running arrears is transferred to the other sector is an indirect way of introducing trade credit repayment.²⁵ The implicit interest on trade credit in this case would depend on the parameters of the penalty function.²⁶

2.4.1 The model with symmetric credit rationing. Under credit rationing $m_{1t} = m_{1t-1} + cr_{1t}$, where $cr_{1t} = Cr_{1t}/p_t$, and the interest rate does not affect producers' decisions. Applying the first sector price indexation rule (9) which, being traced to the initial period $t = 0$, becomes $q_t = (1 + \pi_{t-1})q_{t-1} = \dots = p_{t-1}\tilde{q}_0$, the first sector optimization problem can be written as:

$$\begin{aligned} \max_{x_t, z_{1t}} & \beta_t \tilde{q}_0 b(x_t - az_{1t}^{y+1} m_{1t}^{-y})^\alpha - (1 + \pi_t) k(x_t^{1+\sigma} z_{1t}^{-\sigma} - x_t^\sigma z_{1t}^{1-\sigma}) - (1 + \pi_t) z_{1t}, \\ \text{s.t. } & \Delta m_{1t} = cr_{1t}, \quad m_{1t} = m_{1t-1} + \Delta m_{1t}. \end{aligned} \quad (10)$$

Symmetrically, the second sector solves (with indexation rule (9)):

$$\begin{aligned} \max_{y_t, z_{2t}} & \gamma_t \frac{(1 + \pi_t)(1 + \pi_{t-1})}{\tilde{q}_0} b(y_t - az_{2t}^{y+1} m_{2t}^{-y})^\alpha - \\ & - (1 + \pi_{t-1}) k(y_t^{1+\sigma} z_{2t}^{-\sigma} - y_t^\sigma z_{2t}^{1-\sigma}) - (1 + \pi_{t-1}) z_{2t}, \\ \text{s.t. } & \Delta m_{2t} = cr_{2t}, \quad m_{2t} = m_{2t-1} + \Delta m_{2t}. \end{aligned} \quad (11)$$

The first order conditions for (10) and (11) are easily derived along the lines discussed in Appendix C.

To take into account the existence of two separate monetary circuits in the economy, assume that there is a one-period lag before profits become household income. Assuming also that there are no transaction

²⁵ Here we assume that trade credit debt is very short-term (less than a period), and a certain amount of it (equal to the amount of the penalty) is repaid at the end of the period.

²⁶ Under the parametrization described, the implicit interest rate on trade credit is equal to $\{pk(x/z_1)^\sigma - 1\}$. With low k the implicit interest rate on trade credit may be negative.

costs of converting non-cash into cash²⁷, the consumer's problem in the dynamic case becomes:

$$\begin{aligned} & \max_{c_t, m_t} U(c_t, m_t), \\ & \text{s.t. } p_t c_t + M_t = I_t, \quad m_t = M_t/p_t, \\ & I_t = (1 - \theta)(\varepsilon_{t-1} q^0 \zeta_{t-1} + \gamma_{t-1} p_{t-1} d_{t-1}) + M_{t-1}. \end{aligned} \quad (12)$$

We should mention here that the way we introduced two circuits²⁸ in the model implies a new source of finance for government expenditures g , which are covered now via profit tax revenues, seigniorage, and intermediary's profit.

Table 2. Signs of change of main variables in response to parameters' variation (steady-state comparative statics). First indexation rule.

Variable Parameter	π	x	z_1	f	y	z_2	h	c
g	+	−	−	−	+	+	+	−
θ	−	+	+	+	−	−	−	−
k								
Model with symmetric sectors								
cr_1	+	?↑	?↑	?↑	+	+	+	
cr_2	−	+	+	+	?↑	?↑	?↑	+
Model with asymmetric sectors								
cr_2	−	+	+	+	?↑	?↑	?↑	+
ρ	−	?↓	?↓	?↓	−	−	−	

?↑ means that the influence is likely to be positive, provided that the effect of relaxing the credit limit is stronger than the inflation effect.

?↓ means that the influence is likely to be negative, provided that the interest rate effect is stronger than the inflation effect.

The long-run equilibrium of the model is a steady-state trajectory (*i.e.*, the equilibrium trajectory on which real variables do not change over time), and prices change at a constant rate (*i.e.*, the inflation rate is constant: $\pi_t = \pi_{t+1} = \pi$). Table 2 summarizes the results of the com-

²⁷ The losses due to price changes between periods, "intermediator's profit", are an exception.

²⁸ Enterprises are allowed to use only non-cash, while households need cash.

parative static exercises (the last two rows are relevant for the model with asymmetric sectors).

The long-run equilibrium trajectories depend significantly upon the exogenous variables' indexation rules. If the first sector price is altered in line with the second sector price inflation (according to (9')), then $q_t = (1 + \pi_t)q_{t-1} = p_t \tilde{q}_0$.

Then the energy sector problem, for example, becomes:

$$\begin{aligned} \max_{x_t, z_{1t}} & \beta_t p_t \tilde{q}_0 b(x_t - a z_{1t}^{v+1} m_{1t}^{-v})^\alpha - p_t k(x_t^{1+\sigma} z_{1t}^{-\sigma} - x_t^\sigma z_{1t}^{1-\sigma}) - p_t z_{1t}, \\ \text{s.t. } & \Delta m_{1t} = c r_{1t}, \quad m_{1t} = m_{1t-1} + \Delta m_{1t}. \end{aligned} \quad (13)$$

As a result, the FOCs for (13) (and for the second sector's problem as well) are independent of inflation, and thereby the supply and export functions do not depend on inflation whereas demand still does.

The steady-state comparative statics results under the second indexation rule (according to (9')) are presented in Table A2 in the Appendix (the last two rows are relevant for the model with asymmetric sectors).

2.4.2 The Model with Asymmetric Credit Rationing. Let us suppose that the first (energy) sector is not bound by a credit rationing constraint, which would mean that the demand for credit from the sector is governed by the real interest rate (among other factors), whereas credit to the second (manufacturing) sector is rationed. Expectations are important in this setting. For simplicity let us assume perfect foresight expectations: $\pi_{t+1}^e = \pi_{t+1}$.

In this case the energy sector problem (long-run version) can be written as:

$$\begin{aligned} \max_{x_t, \Delta m_{1t}, z_{1t}} & \beta_t q_t b(x_t - a z_{1t}^{v+1} m_{1t}^{-v})^\alpha - \\ & - p_t k(x_t^{1+\sigma} z_{1t}^{-\sigma} - x_t^\sigma z_{1t}^{1-\sigma}) - p_t z_{1t} - R_{t+1}^e p_t \Delta m_{1t}, \\ R_{t+1}^e &= (1 + \rho)(1 + \pi_{t+1}^e) - 1, \\ \Delta m_{1t} &< C r_{1t} / p_t, \quad m_{1t} = m_{1t-1} + \Delta m_{1t}. \end{aligned} \quad (14)$$

Applying the first indexation rule (9) and dividing by p_t , we obtain:

$$\begin{aligned} \max_{x_t, z_{1t}, \Delta m_{1t}} & \frac{\beta_t \tilde{q}_0 b}{(1 + \pi_t)} (x_t - a z_{1t}^{v+1} m_{1t}^{-v})^\alpha - \\ & - k(x_t^{1+\sigma} z_{1t}^{-\sigma} - x_t^\sigma z_{1t}^{1-\sigma}) - z_{1t} - (\rho + \pi_{t+1} + \rho \pi_{t+1}) \Delta m_{1t}, \\ \text{s.t. } & \Delta m_{1t} < c r_{1t}, \quad m_{1t} = m_{1t-1} + \Delta m_{1t}. \end{aligned} \quad (15)$$

The second (manufacturing) sector's problem (under the first indexation rule) stays the same:

$$\begin{aligned} \max_{y_t, z_{2t}} & \gamma_t \frac{(1 + \pi_t)(1 + \pi_{t-1})}{\tilde{q}_0} b(y_t - a z_{2t}^{v+1} m_{2t}^{-v})^\alpha - \\ & - (1 + \pi_{t-1}) k(y_t^{1+\sigma} z_{2t}^{-\sigma} - y_t^\sigma z_{2t}^{1-\sigma}) - (1 + \pi_{t-1}) z_{2t}, \\ \text{s.t. } & \Delta m_{2t} = c r_{2t}, \quad m_{2t} = m_{2t-1} + \Delta m_{2t}. \end{aligned} \quad (16)$$

The supply function and demand in steady state depend on parameters in the following manner:

$$\begin{aligned} S &= S((1 + \pi) / \tilde{q}_0, \beta, \gamma, \rho, c r_2, k), \\ & \quad + \quad ? \quad ? \quad + \quad + \quad ? \\ \xi &= \xi((1 + \pi) / \tilde{q}_0, \beta, \gamma, \rho, c r_2, k), \\ & \quad - \quad ? \quad ? \quad - \quad - \quad ? \\ D &= D(g, (1 + \pi) / \tilde{q}_0, \tilde{e}_0 / \tilde{q}_0, \Theta, \gamma, \beta, \rho, c r_2). \\ & \quad + \quad - \quad + \quad - \quad ? \quad ? \quad - \quad - \end{aligned} \quad (17)$$

The above (17) suggests that the comparative static results will be the same as in the case of credit rationing of the two sectors, except that the real interest rate change will now have real effects. The results of the steady-state comparative statics in the asymmetric sectors framework can be found in Table 2 (the first indexation rule) and Table A2 (the second indexation rule).

3. DISCUSSION OF RESULTS

The steady-state comparative statics suggests the following:

- An increase in government expenditures, though increasing the second sector's output, results in a steady state with higher inflation and diminished first sector production and export. The result is sensitive to the price indexation rule: if the second rule is in operation, the outcome would be increased inflation and crowding out of consumption, while both sectors' levels of production and export would stay the same.
- An increase in the tax rate stimulates the first sector's production and exports, and represses inflation. At the same time, the second sector's output would diminish. If the second indexation rule is applied, real variables would not change, while inflation would fall.
- If the credit limit to the first sector is relaxed (in the symmetric sectors model), the new steady state would be characterized by higher inflation and lower consumption of the domestic good. At the same time, the second sector's production would expand and, provided that the credit rationing constraint is very restrictive and thereby the effect of its relaxation large compared to the price change influence²⁹, the first sector's output would rise as well. The effect on exports is ambiguous. The result is sensitive to the indexation rule: under the second rule, the energy sector's production would expand, and exports would increase, while the manufacturing sector's output would not change; inflation would still accelerate, and consumption would diminish (as under the first indexation rule).
- A relaxation of the credit limit to the second sector (in the symmetric sectors case) would give the following results: the new steady state inflation would be lower, consumption of the domestic good would be higher, energy sector output would expand, and the second sector's output would possibly increase as well, provided that the effect of relaxing the credit limit outweighs the influence of the inflation ratio. The changes in exports would be ambiguous. The result is again sensitive to the indexation formula: if the second indexation rule is applied, the second sector's output would expand, while exports would contract since there would be no change in the first sector's production; inflation would still decrease and consumption of the domestic good would still expand.

²⁹ If, additionally, demand is rather elastic with respect to price changes, the change in the price ratio itself would not be large.

- A relaxation of the second sector credit limit in the model with asymmetric sectors gives results similar to those mentioned above. The only difference is that, under the first indexation rule, exports are likely to expand, provided that the effect of relaxing the credit limit outweighs the influence of the inflation ratio.
- An increase in real interest rates would lead to a decline in inflation and increased consumption of the domestic good. At the same time, the second sector's production would fall, and the first sector's output and exports are likely to contract as well, provided that the interest rate effect is stronger than the inflation ratio's influence. Under the second indexation rule, there would be a decline in the first sector's production and exports, while there would be no change in the second sector. Inflation would decrease, and consumption of the domestic good would increase (as under the first indexation rule).

The first two comparative statics results are the same as in (Granville *et al.* (1996)), though the mechanisms are slightly different, since the optimal payment ratios of the two sectors, γ^* and β^* , depend now (under credit rationing) not only on the interest rate as in (Granville *et al.* (1996)), but also on each other as well as the price ratio, real money balances and the penalty coefficient. The latter results are quite original (the interest rate influence was ambiguous in the model adopted in Granville *et al.* (1996)).

Overall, it is likely that monetary policy, along with other factors, has added to the depth of the observed recession in Russia from 1992 to 96. The sectoral pattern of the recession seems to have been influenced by the monetary impulse as well. The role of the CBR during the first years of transformation, when the new banking system was weak and unable to create credit resources on a scale required by an economy with a high share of industrial production, continues to be dominant. The transfer of responsibilities to the weak financial system resulted in a lack of liquid financial assets in the system. This was later translated into an extremely high interest rate on money resources and the outburst of quasi-money.

Interenterprise arrears, being a "cushion" for enterprises in periods of tight liquidity, but not a perfect substitute for banking credit, complicate the monetary impulse propagation. As the two-sector model shows, interenterprise arrears are significant in the transmission mechanism: when the credit limit to a sector is relaxed, the sector's payment ratio increases, thus facilitating the other sector's production. The resulting steady-state inflation is not symmetric across the sectors: it increases if

the first (energy) sector receives additional credit, and decreases if the credit is extended to the consumption good producer.

The economic intuition behind the sectoral asymmetry in terms of the inflation that results from the relaxation of the credit limit is likely to be related to the changes in the sources that finance government expenditure. According to the dynamic analogue of (5), government consumption in the steady state is:

$$g = d \left\{ 1 - \frac{(1-\theta)\gamma}{(1+\pi)} \right\} + \frac{(\theta-1)\tilde{\varepsilon}_0 q^0}{(1+\pi)^2} \xi + \frac{\pi}{(1+\pi)} \Delta m.$$

When the credit limit to the first sector is relaxed (and inflation is not changed yet), the payment ratio of the first sector (γ) increases, and hence, demand for the input from the sector increases. The increase in x causes the supply $d = \{h - x\}$ to contract. As a result, tax revenues decrease. Given the level of government expenditure g , the inflation tax revenue substitutes for the reduction of "normal" tax finance. Given the level of m^{30} , the steady-state inflation rate needs to be increased. Schematically:

$$\begin{aligned} cr_1 \uparrow &\Rightarrow \gamma \uparrow \Rightarrow x \uparrow, f \uparrow \Rightarrow d = \{h - x\} \downarrow \Rightarrow \text{tax revenue} \downarrow \Rightarrow \\ &\text{given the level of } g, \\ &\text{the inflationary tax should be increased} \Rightarrow \\ &\Rightarrow \text{given the level of } m, \text{ inflation } \pi \uparrow. \end{aligned}$$

When the credit limit to the second sector is relaxed (and inflation is not changed yet), the payment ratio of the second sector (β) increases, and hence, output of the sector increases. The increase in h causes the supply $d = \{h - x\}$ to expand. As a result, tax revenue increases. Given the level of government expenditure g , the inflation tax revenue may be reduced. Given the level of m , the steady-state inflation rate may be diminished. Schematically,

$$\begin{aligned} cr_2 \uparrow &\Rightarrow \beta \uparrow \Rightarrow y \uparrow, h \uparrow \Rightarrow d = \{h - x\} \uparrow \Rightarrow \text{tax revenue} \uparrow \Rightarrow \\ &\Rightarrow \text{given the level of } g, \\ &\text{the inflationary tax should be decreased} \Rightarrow \\ &\Rightarrow \text{given the level of } m, \text{ inflation } \pi \downarrow. \end{aligned}$$

It is shown in Appendix D that there is no difference in the direction of change (under parameters' variation) in the model with payment arrears

³⁰ Demand for real money balances from consumers is likely to decrease given a decrease in income (which depends on d).

relative to the model without arrears. However, the magnitude of the initiated changes in the two models may be different given the differences in the mechanisms of transmission. It turns out that interenterprise arrears may soften or strengthen the supply side reaction in the steady state, depending on certain elasticities (see Appendix for details). The latter is likely to depend on the characteristics of the production and transaction cost functions and the level of real money balances. Empirical evidence, suggesting that money elasticity of arrears ratio (for both sectors) is less than one (Denisova (1997)) appears to favor the "cushioning" effect of arrears during the period.

How might we interpret the model's result that it is possible to simultaneously decrease inflation and raise output in light of the Phillips curve tradeoff? The credit crunch, exerting an adverse influence on the supply side of the economy, seems to have shifted the level of potential (long-run equilibrium) output. As a result, the locus of the inflation-unemployment tradeoff (the Phillips curve) has shifted as well.³¹ When the credit limit to the manufacturing sector is relaxed, the supply curve shifts rightwards (reflecting an increase in potential output), which corresponds to a leftward shift of the Phillips curve. Thus, in the absence of (essentially inflationary) accommodative demand management policy, the steady-state inflation may be reduced, while long-run equilibrium output can increase. The sectoral asymmetry (in terms of inflation) of the result seems to originate from the corresponding difference in government expenditure finance discussed above, and is on the demand side of the conventional model.

Hence, the model provides an opportunity to trace the influence of different policies on real production and inflation in the complex environment of a transitional economy. In particular, by emphasizing the transaction role of liquid assets, the framework allows one to consider the real effects of monetary tightening and the shift to quasi-money (arrears). Distinguishing between the two sectors helps one to clarify those factors that determine the dynamics of the sectoral pattern of output. Moreover, the model suggests that the depth of the recession in Russian industry, and its sectoral pattern, may (along with other factors) be explained by the policy of excessive credit rationing, especially of the second sector.³² The model highlights the mechanisms through which

³¹ The subsequent shifts of the curve (following the changes in inflationary expectations, *e.g.*) are going to be "around" the new level of potential output.

³² The energy sector, having strong bargaining power in negotiations with the government, is known to have better access to banking credit and government subsidies. Real credit to the sector was increasing in 1994 – 1996.

policy, meant to fight inflation, could cause increased inflation together with a decline in both sectors' output and consumption.

The first (symmetric) version of the model seems to reflect the realities of the Russian economy in 1992 – 1993, while the second (asymmetric) version may be useful for discussing later developments. The model suggests that a relaxation of the credit rationing constraint to the second sector, and/or a decrease in interest rates, may be growth promoting strategies, and the former may not even result in higher inflation, implying that there is scope for governmental intervention.

APPENDICES

A. Tables

Table A1. Short-term (liquid) assets and liabilities of industrial enterprises (% of total short-term liabilities, end of month).

Assets				Liabilities			
	Nov-93	Nov-94	Nov-95		Nov-93	Nov-94	Nov-95
Total industry							
Total debtors	74.3	70.3	61.8	Total creditors	83.7	79.9	82.3
<i>incl. in arrears</i>	30.2	38.3	29.0	<i>incl. in arrears</i>	29.2	42.7	41.8
Receivables from customers	60.7	57.0	48.0	Payables to suppliers	52.3	48.7	43.3
<i>incl. overdue</i>	26.1	33.0	24.9	<i>incl. overdue</i>	21.3	27.6	19.8
Cash and bank deposits	11.2	6.8	3.1	Payables to the budget	11.1	14.4	17.6
Financial investment	2.4	5.3	7.6	<i>incl. overdue</i>	3.7	8.9	11.6
				Total bank credit	16.3	20.1	17.7
				<i>incl. overdue</i>	1.2	2.4	2.0
Energy sector*							
Total debtors	80.2	86.5	77.2	Total creditors	95.0	93.8	92.1
<i>incl. in arrears</i>	33.2	54.3	43.0	<i>incl. in arrears</i>	30.0	56.9	54.1
Receivables from customers	63.9	72.0	63.7	Payables to suppliers	55.5	57.9	49.4
<i>incl. overdue</i>	28.1	47.0	37.5	<i>incl. overdue</i>	20.6	35.8	25.8
Cash and bank deposits	16.9	10.7	2.7	Payables to the budget	19.6	21.4	23.1
Financial investment	2.5	6.7	9.6	<i>incl. overdue</i>	6.4	14.9	17.2
				Total bank credit	4.9	6.2	7.9
				<i>incl. overdue</i>	0.2	0.5	0.5

Source: Author's calculations based on Goskomstat data.

* The energy sector is defined to include electricity and fuel sector.

Table A1 continued from p. 35

Assets				Liabilities			
	Nov-93	Nov-94	Nov-95		Nov-93	Nov-94	Nov-95
Manufacturing sector*							
Total debtors	70.2	60.7	52.9	Total creditors	78.2	71.8	76.9
<i>incl. in arrears</i>	28.5	29.1	21.0	<i>incl. in arrears</i>	28.5	33.4	35.2
Receivables from customers	58.1	47.9	40.3	Payables to suppliers	48.4	42.7	39.1
<i>incl. overdue</i>	24.9	25.0	17.7	<i>incl. overdue</i>	21.3	21.7	16.3
Cash and bank deposits	7.8	4.4	3.3	Payables to the budget	6.7	10.6	14.8
Financial investment	2.3	4.6	6.6	<i>incl. overdue</i>	2.3	5.6	8.6
				Total bank credit	21.8	28.2	23.1
				<i>incl. overdue</i>	1.7	3.6	2.9

Source: Author's calculations based on Goskomstat data

* The manufacturing sector encompasses the rest eight industrial branches.

Table A2. Signs of change of main variables in response to parameters' variation (steady-state comparative statics). Second indexation rule.

Variable Parameter	π	x	z_1	f	y	z_2	h	c
g	+	no change	no change	no change	no change	no change	no change	–
θ	–	no change	no change	no change	no change	no change	no change	no change
k								
Model with symmetric sectors								
cr_1	+	+	+	+	no change	no change	no change	
cr_2	–	no change	no change	no change	+	+	+	+
Model with asymmetric sectors								
cr_2	–	no change	no change	no change	+	+	+	+
ρ	–	–	–	–	no change	no change	no change	

B. List of variables used in the model

Small letters denote variables in real terms and commodity prices; capital letters denote nominal variables:

- q — domestic price for sector 1 product;
- p — domestic price for sector 2 product;
- x — the first sector's demand for working capital;
- z_1 — the first sector's demand for paid for working capital;
- \bar{x} — the first sector's working capital actually used (*i.e.* x adjusted for transactional costs);
- m_1 — the first sector's demand for real money balances ($m_1 = M_1/q$, where M_1 are the first sector nominal money balances);
- f — the first sector's production for domestic and foreign markets (supply) $f(x, m_1, z_1) = \Omega(\bar{x}) = \Omega(x - \mu(z_1/m_1)z_1)$.

The production function Ω , transaction costs function μ and penalty function ϕ are assumed to be of the following form:

$$\Omega(\bar{x}) = b\bar{x}^\alpha, \quad \mu(m_1/z_1) = a(m_1/z_1)^\nu, \quad \phi(x/z_1) = k(x/z_1)^\sigma, \\ \alpha \in (0,1), \quad \sigma \in (0,1).$$

- ξ — exports;
- y — the second sector's demand for working capital;
- z_2 — the second sector's demand for paid for working capital;
- \bar{y} — the second sector's working capital actually used (*i.e.* y adjusted for transactional costs);
- m_2 — the second sector's demand for real money balances ($m_2 = M_2/p$, where M_2 are the second sector nominal money balances);
- h — the second sector's production (supply)

$$h(y, m_2, z_2) = \Omega(\bar{y}) = \Omega(y - \mu(m_2/z_2)z_2), \\ \Omega(\bar{y}) = b\bar{y}^\alpha, \quad \mu(m_2/z_2) = a(m_2/z_2)^\nu, \quad \phi(y/z_2) = k(y/z_2)^\sigma;$$

- c — consumer demand for the domestic consumption good;
- m — consumer demand for real money balances ($m = M/p$, where M are consumer's nominal money balances);
- M_0 — consumer's money from previous period in nominal terms;

- cr_1 — credit limit for the first sector ($cr_1 = Cr_1/p$, where Cr_1 is the first sector limit for banking credit in nominal terms);
- cr_2 — credit limit for the second sector ($cr_2 = Cr_2/q$, where Cr_2 is the second sector limit for banking credit in nominal terms);
- R — nominal interest rate;
- ρ — real interest rate;
- θ — profit tax rate;
- g — government demand for sector 2 product;
- d — government and consumer's total demand for sector 2 product;
- γ — the first sector payment ratio ($1 - \gamma$ is the first sector payables (due to the second sector) ratio);
- β — the second sector payment ratio ($1 - \beta$ is the second sector payables (due to the first sector) ratio);
- π — the second good prices' inflation rate.

C. First-order conditions

With the parameterization and under the credit rationing regime, the first sector (energy producer) problem can be stated as:

$$\begin{aligned} \max_{x, z_1} \quad & \beta q b (x - a z_1^{v+1} m_1^{-v})^\alpha - p k (x^{1+\sigma} z_1^{-\sigma} - x^\sigma z_1^{1-\sigma}) - p z_1 - R p c r_1, \\ \text{s.t.} \quad & \Delta m_1 = c r_1, \quad m_1 = m_{10} + \Delta m_1. \end{aligned} \quad (18)$$

Note that the last term in the objective function is constant and hence can be omitted.

Differentiating with respect to x and z_1 , one obtains the first order conditions:

$$\alpha \beta q b (x - a z_1^{v+1} m_1^{-v})^{\alpha-1} - p k (1 + \sigma) x^\sigma z_1^{-\sigma} + p k \sigma x^{\sigma-1} z_1^{1-\sigma} = 0, \quad (19)$$

$$\begin{aligned} \alpha \beta q b (x - a z_1^{v+1} m_1^{-v})^{\alpha-1} (-a)(v+1) z_1^v m_1^{-v} + \\ + p k \sigma x^{1+\sigma} z_1^{-\sigma-1} + p k (1 - \sigma) x^\sigma z_1^{-\sigma} - p = 0, \end{aligned} \quad (20)$$

$$\Delta m_1 = c r_1, \quad m_1 = m_{10} + \Delta m_1. \quad (21)$$

Define the following

$$\begin{aligned} \gamma = z_1 / x, \quad A = \beta q b / p, \quad \psi_0(x, z_1, m_1, A) = A \alpha (x - a z_1^{1+v} m_1^{-v})^{\alpha-1}, \\ V(\gamma) = (1 + \sigma) \gamma^{-\sigma} - \sigma \gamma^{1-\sigma}, \quad W(\gamma) = \sigma \gamma^{-1-\sigma} + (1 - \sigma) \gamma^{-\sigma} - 1/k. \end{aligned} \quad (22)$$

Then we can rewrite the FOC in the form:

$$\psi_0 = kV(\gamma), \quad (19')$$

$$\psi_0 = k(a(1+\nu))^{-1} m_1^\nu z_1^{-\nu} W(\gamma), \quad (20')$$

$$\Delta m_1 = cr_1, \quad m_1 = m_{10} + \Delta m_1, \quad (21')$$

γ is the ratio of sector 1 input, which is paid for, to total demanded input, and thus $(1 - \gamma)$ is the arrears' ratio of sector 1.

It is easy to see that $V(\gamma)$ and $W(\gamma)$ are decreasing functions of γ , provided $\gamma < 1$.

Putting conditions (19') and (20') together allows one to obtain the following expression for z_1 :

$$z_1 = (V(\gamma)/W(\gamma))^{-1/\nu} a^{-1/\nu} (1+\nu)^{-1/\nu} m_1. \quad (23)$$

From (19') we have: $x = (kV(\gamma)/A\alpha)^{-1/(1-\alpha)} (1 - a\gamma m_1^{-\nu} z_1^\nu)^{-1}$.

Substituting z_1 from (23), we get:

$$x = (kV(\gamma)/A\alpha)^{-1/(1-\alpha)} \left(1 - a\gamma (V(\gamma)/W(\gamma))^{-1} a^{-1} (1+\nu)^{-1}\right)^{-1}. \quad (24)$$

Combining (23) and (24) provides an equation for γ :

$$\Phi(\gamma) = 0,$$

$$\begin{aligned} \Phi(\gamma) \equiv & \gamma - \left\{ \frac{V(\gamma)}{W(\gamma)} a(1+\nu) \right\}^{-1/\nu} m_1 (kV(\gamma)/A\alpha)^{1/(1-\alpha)} + \\ & + a\gamma \left\{ \frac{V(\gamma)}{W(\gamma)} a(1+\nu) \right\}^{-(\nu+1)/\nu} m_1 (kV(\gamma)/A\alpha)^{1/(1-\alpha)}. \end{aligned} \quad (25)$$

Solving (25) for γ results in the value of γ , γ^* , that maximizes the first sector objective function, and the optimal amounts of input demanded and paid for, *i.e.* x^* and z_1^* respectively. It is proved in Appendix E1 that $\Phi(\gamma)$ is an increasing function of γ , *i.e.* $\partial\Phi/\partial\gamma > 0$, provided that the elasticity of $V(\gamma)/W(\gamma)$ with respect to γ is less than one. To guarantee that $\gamma^* < 1$ (which means that it is optimal for the sector not to pay for its input in full), we need to impose the following constraint on the parameters:

$$1 + \frac{1-1/k}{1+\nu} > \frac{1}{m_1 \left((1-k^{-1})/a(1+\nu) \right)^{1/\nu} (K/A\alpha)^{1/(1-\alpha)}}. \quad (26)$$

The inequality (26) comes from the fact that $\Phi(\gamma = 1) > 0$, which, together with $\partial\Phi/\partial\gamma > 0$, guarantees that $\gamma^* < 1$. One may notice that, as the second element on the left side of (26) is less than 0 ($0 < k < 1$), when k is small (closer to 0 than to 1), the condition requires m_1 to be rather small. This implies that real money balances would have to fall below a certain critical level to switch on arrears (provided that the penalty is not large). This is consistent with the observation that the sharp contraction of real money balances in 1992, caused by the initial price jump and the strict credit policy, initiated the shift into quasi-money (arrears' obligations).

It is clear from (25) that $\Phi(\gamma)$ depends on the parameters $m_1 [= m_{10} + cr_1]$, $A [= \beta qb/p]$ and k , and therefore γ is a function of cr_1 , β , p/q and k . It is proved in the Appendix that the first sector payment ratio γ is a decreasing function of the second sector payment ratio (β) and an increasing function of the price ratio (p/q), the first sector credit limit in real terms (cr_1) and the penalty function coefficient (k), or the implicit interest rate on trade credit.³³

The first sector input paid for (z_1) is a decreasing function of γ , while the input demanded (x) is an increasing function of γ . As a result, the first sector demand for working capital, paid for input and output increase when the second sector payment ratio increases or the first sector credit limit is relaxed, and decrease when the price ratio deteriorates for the sector or the penalty for having arrears increases.

The second (manufacturing) sector problem is symmetric to the first sector's if we assume:

$$\Omega(\bar{y}) = b\bar{y}^\alpha, \quad \mu(m_2/z_2) = a(m_2/z_2)^v, \quad \phi(y/z_2) = k(y/z_2)^\sigma.$$

With the parameterization, the manufacturing sector problem can be stated as:

$$\begin{aligned} \max_{y, \Delta m_2, z_2} \quad & \gamma pb(y - az_2^{v+1}m_2^{-v})^\alpha - qk(y^{1+\sigma}z_2^{-\sigma} - y^\sigma z_2^{1-\sigma}) - \\ & - qz_2 - Rq\Delta m_2, \\ \text{s.t.} \quad & \Delta m_2 \leq Cr_2/q, \quad m_2 = m_{20} + \Delta m_2. \end{aligned} \quad (27)$$

³³ The higher the penalty function coefficient, the higher the implicit interest rate on trade credit, i.e. $(pk(1/\gamma)^\sigma - 1)$ in terms of the first sector and $(qk(1/\beta)^\sigma - 1)$ in terms of the second sector.

The first order conditions for the second sector maximization problem (when credit rationing is binding) are symmetric to the first sector FOCs:

$$\begin{aligned}\tilde{\psi}_0 &= k a^{-1} (1 + v)^{-1} m_2^v z_2^{-v} \tilde{W}(\beta), \\ \Delta m_2 &= c r_2, \quad m_2 = m_{20} + \Delta m_2,\end{aligned}\tag{28}$$

where $\beta = z_2 / y$, $\tilde{A} = \gamma p b / q$, $\tilde{\psi}_0(y, z_2, m_2, \tilde{A}) = \tilde{A} \alpha (y - a z_2^{1+v} m_2^{-v})^{\alpha-1}$, $\tilde{V}(\beta) = (1 + \sigma) \beta^{-\sigma} - \sigma \beta^{1-\sigma}$, $\tilde{W}(\beta) = \sigma \beta^{-(1+\sigma)} + (1 - \sigma) \beta^{-\sigma} - 1/k$.

β is the ratio of sector 2 input, which is paid for, to total demanded input, and thus $(1 - \beta)$ is the arrears' ratio of sector 2. To guarantee that $\beta^* < 1$ we need to impose a similar constraint on parameters as in (26). Like the energy sector, manufacturing sector non-payments are switched on by its real money balances being below some critical level (given that the penalty coefficient is not large).

Similar to the procedure used above, we can establish that β is a decreasing function of the first sector payment ratio (γ) and the price ratio (p/q), and an increasing function of the credit limit for the second sector in real terms ($c r_2$) and the penalty function coefficient (k).

Table C1 summarizes the main results of the partial equilibrium analysis.

The first order conditions for the consumer's maximization problem are standard. Assuming that goods are normal, and given the gross substitutability assumption, the resulting demand functions for the domestic consumption good c and real money balances m depend negatively on their own price and positively on income.

Given the assumptions made about the functions f , h , ϕ , μ and U , and assuming that substitution effect caused by the price ratio change is stronger than the income effect³⁴, an equilibrium exists and is unique.

D. Two-sector model with credit rationing (without arrears)

Suppose that there are still transaction costs of buying working capital, but there is no possibility to hold payment arrears (*e.g.*, they are strictly prosecuted legally³⁵), so that now (in terms of the first sector) $x = z_1$

³⁴ This would guarantee that the demand function defined below depends negatively on the price ratio p/q .

³⁵ Another way to introduce the impossibility of running payment arrears is to make $\sigma \rightarrow \infty$ (*i.e.* to drive the penalty to infinity), so that again $x = z_1$ and $y = z_2$.

Table C1. Signs of partial derivatives of main variables with respect to parameters (partial equilibrium analysis).

Variable Parameter	x	z ₁	\bar{x}	f	y	z ₂	\bar{y}	h	ξ	d ^s
γ	+	-	+	+	+	+	+	+	?	?
β	+	+	+	+	+	-	+	+	?	?
p/q	-	-	-	-	+	+	+	+	-	+
cr ₁	+	+	+	+					+	-
cr ₂					+	+	+	+	-	+
k	-	-	-	-	-	-	-	-	?	?

and actual demand for input is $\bar{x} = x - \mu(x/m_1)x$. In this case (and under binding credit rationing) the first sector problem can be written as:

$$\begin{aligned} \max_{x_t} & q_t b(x_t - ax_t^{v+1} m_{1t}^{-v})^\alpha - p_t x_t, \\ \text{s.t. } & \Delta m_{1t} = cr_{1t}, \quad m_{1t} = m_{1t-1} + \Delta m_{1t}. \end{aligned} \quad (29)$$

Actual demand for input is now $\bar{x}_t = x_t - \mu(x_t/m_{1t})x_t$.

The first order conditions of the problem (under the first indexation rule) are:

$$\alpha \tilde{q}_0 b x_t^{\alpha-1} (1 - ax_t^v m_{1t}^{-v})^{\alpha-1} (1 - ax_t^v m_{1t}^{-v} + x_t) - (1 + \pi_t) = 0, \quad (30)$$

$$\Delta m_{1t} = cr_{1t}, \quad m_{1t} = m_{1t-1} + \Delta m_{1t}. \quad (31)$$

It can be shown that (under standard assumptions about the production function, which are translated here into restrictions on parameters) the demand for working capital and output of the first sector are positive functions of the credit limit to the sector and negative functions of the inflation ratio $(1 + \pi)/\tilde{q}_0$. Symmetrically, the second sector's demand for working capital and output are positive functions of the credit limit to the second sector and of the inflation ratio.

As a result, the supply function depends on parameters in the following way:

$$S = S((1 + \pi)/\tilde{q}_0, cr_1, cr_2). \quad (32)$$

$\begin{matrix} + & - & + \end{matrix}$

It can be shown that demand depends on parameters in the following fashion:

$$D = D(\underset{+}{g}, \underset{-}{(1 + \pi)} / \underset{+}{\tilde{q}_0}, \underset{-}{\tilde{\epsilon}_0} / \underset{+}{\tilde{q}_0}, \underset{+}{\Theta}, \underset{-}{cr_1}, \underset{-}{cr_2}). \quad (33)$$

Hence, there is no difference in the direction of change (under parameters' variation) in comparison with the model with arrears. However, in the model without arrears, a credit limit relaxation directly affects the (actual) demand for working capital, and therefore, output, *i.e.*

$$\frac{\partial f}{\partial cr_1} = \frac{\partial f}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial cr_1}$$

(in the first sector variables). In the model with payment arrears the effect is indirect: a change in the credit limit, and thus real money balances, is translated into a changed level of the sector's payment ratio, and the payment ratio change affects the demand for working capital and output, *i.e.*

$$\frac{\partial f}{\partial cr_1} = \frac{\partial f}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial \gamma} \frac{\partial \gamma}{\partial cr_1}.$$

Therefore, the comparative strength of initiated (by changes in credit limit) supply-side reactions in the two models depends on the magnitudes of the respective elasticities. The latter is likely to depend on the characteristics of the production and transaction cost functions, and the level of real money balances. It is difficult to draw an unambiguous conclusion about the comparative magnitude of the respective elasticities analytically.

However, one may expect arrears to exert a "cushioning" effect on the supply-side reaction. In the presence of transaction costs, a decrease in the credit limit to a sector (following monetary tightening) causes a decline in the actual demand for input \bar{x} . When there is no alternative source of liquidity (which would lessen transaction costs) the resulting decrease in \bar{x} , and therefore f , may be high. Payment arrears provide a way to diminish transaction costs, and therefore allow to counteract the adverse influence of credit tightening. Hence, the resulting decrease in \bar{x} , and therefore f , is likely to be less than in the absence of arrears. The result is true in a certain region where the costs of increasing arrears (*i.e.*, the penalties) are still less than the benefits (*i.e.*, the decrease in transaction costs).

An increase in the credit limit to a sector results in the increase of the actual demand for input \bar{x} (via decreasing transaction costs). In the presence of arrears, the relaxation of the credit limit to a sector in-

creases the payment ratio, which in turn increases the actual demand for input \bar{x} . It is likely that when the payment ratio is very low, the relaxation of the credit limit may cause a substantial increase in the payment ratio, and consequently a substantial increase in output. When γ is high, the resulting change in the payment ratio may not be high.³⁶

Hence, given the non-linearities of the relationships under consideration, it is difficult to draw an unambiguous conclusion about the comparative magnitude of the initiated supply side reaction analytically. If the elasticity of payment arrears with respect to credit is less than one, *i.e.* (in terms of the first sector variables)

$$\frac{\partial \gamma}{\partial cr_1} \frac{cr_1}{\gamma} < 1,$$

one may expect arrears to have a "cushioning" effect. Empirical analysis may help in this respect.

E. Proofs

E1. Let us prove that $\partial \Phi / \partial \gamma > 0$.

$$\begin{aligned} \frac{\partial \Phi}{\partial \gamma} = & \frac{1}{v} \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-(v+1)/v} \frac{\partial(V(\gamma)/W(\gamma))}{\partial \gamma} (a(1+v))^{-1/v} m_1 \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{1/(1-\alpha)} - \\ & - \gamma \frac{1}{v} \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-(2v+1)/v} \frac{\partial(V(\gamma)/W(\gamma))}{\partial \gamma} (a(1+v))^{-(v+1)/v} m_1 \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{1/(1-\alpha)} - \\ & - \frac{1}{1-\alpha} \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-1/v} (a(1+v))^{-1/v} m_1 \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{\alpha/(1-\alpha)} \frac{k}{A\alpha} \frac{\partial V(\gamma)}{\partial \gamma} + \\ & + \gamma \frac{1}{1-\alpha} \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-(v+1)/v} a(a(1+v))^{-(v+1)/v} m_1 \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{\alpha/(1-\alpha)} \frac{k}{A\alpha} \frac{\partial V(\gamma)}{\partial \gamma} + \\ & + 1 + \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-(v+1)/v} a(a(1+v))^{-(v+1)/v} m_1 \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{1/(1-\alpha)} - \\ & - \gamma \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-(2v+1)/v} a(a(1+v))^{-(v+1)/v} m_1 \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{1/(1-\alpha)} \frac{\partial(V(\gamma)/W(\gamma))}{\partial \gamma}. \end{aligned}$$

³⁶ This could be the case (when the payment ratio is high) when increased real money balances make it profitable not to run payment arrears and therefore to pay in full.

It is easy to see that $\partial\Phi/\partial\gamma > 0$, since $\gamma < 1$, $\partial V(\gamma)\Phi/\partial\gamma > 0$, $\partial(V(\gamma)/W(\gamma))/\partial\gamma > 0$, and $1 - \gamma(V(\gamma)/W(\gamma))^{-1}(1+\nu)^{-1} > 0$, and assuming that the elasticity of $V(\gamma)/W(\gamma)$ with respect to γ is less than one, a sufficient condition for which is:

$$\begin{aligned} \gamma \left\{ \frac{\partial(V(\gamma)/W(\gamma))}{\partial\gamma} \right\} \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-1} < \\ < 1 + \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{(1+\nu)/\nu} a^{1/\nu} (1+\nu)^{(1+\nu)/\nu} m_1^{-1} (A\alpha)^{1/(1-\alpha)} (kV(\gamma))^{-1/(1-\alpha)}. \end{aligned}$$

E2. Let us prove that $\partial\Phi/\partial A > 0$, $\partial\Phi/\partial m_1 < 0$, $\partial\Phi/\partial k < 0$.

$$\begin{aligned} \frac{\partial\Phi}{\partial A} = \frac{1}{1-\alpha} \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{\alpha/(1-\alpha)} \frac{kV(\gamma)}{\alpha} A^{-2} \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-1/\nu} (a(1+\nu))^{-1/\nu} m_1 - \\ - \gamma \frac{1}{1-\alpha} \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{\alpha/(1-\alpha)} \frac{kV(\gamma)}{\alpha} A^{-2} \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-(\nu+1)/\nu} a(a(1+\nu))^{-(\nu+1)/\nu} m_1. \end{aligned}$$

Since $1 - \gamma(V(\gamma)/W(\gamma))^{-1}(1+\nu)^{-1} > 0$, $\partial\Phi/\partial A > 0$.

$$\begin{aligned} \frac{\partial\Phi}{\partial m_1} = - \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{1/(1-\alpha)} \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-1/\nu} (a(1+\nu))^{-1/\nu} + \\ + \gamma \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{1/(1-\alpha)} \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-(\nu+1)/\nu} a(a(1+\nu))^{-(\nu+1)/\nu}. \end{aligned}$$

Since $1 - \gamma(V(\gamma)/W(\gamma))^{-1}(1+\nu)^{-1} > 0$, $\partial\Phi/\partial m_1 < 0$.

$$\begin{aligned} \frac{\partial\Phi}{\partial k} = - \frac{1}{1-\alpha} \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{\alpha/(1-\alpha)} \frac{V(\gamma)}{A\alpha} \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-1/\nu} (a(1+\nu))^{-1/\nu} m_1 + \\ + \gamma \frac{1}{1-\alpha} \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{\alpha/(1-\alpha)} \frac{V(\gamma)}{A\alpha} \left\{ \frac{V(\gamma)}{W(\gamma)} \right\}^{-(\nu+1)/\nu} a(a(1+\nu))^{-(\nu+1)/\nu} m_1. \end{aligned}$$

Since $1 - \gamma(V(\gamma)/W(\gamma))^{-1}(1+\nu)^{-1} > 0$, $\partial\Phi/\partial k < 0$.

E3. Let us prove that $\partial x / \partial \beta > 0$, $\partial x / \partial (p/q) < 0$, $\partial x / \partial m_1 > 0$, $\partial x / \partial k < 0$ and $\partial z_1 / \partial \beta > 0$, $\partial z_1 / \partial (p/q) < 0$, $\partial z_1 / \partial m_1 > 0$, $\partial z_1 / \partial k < 0$.

x depends on m_1 only through γ , and on A and k — both through γ and directly. z_1 depends on A and k only through γ , and on m_1 — both via γ and directly. Using the fact that $x = z_1 / \gamma$, $z_1 = x\gamma$ and that

$$\frac{\partial x}{\partial m_1} = \frac{\partial x}{\partial \gamma} \frac{\partial \gamma}{\partial m_1} > 0, \quad \frac{\partial z_1}{\partial A} = \frac{\partial z_1}{\partial \gamma} \frac{\partial \gamma}{\partial A} > 0, \quad \frac{\partial z_1}{\partial k} = \frac{\partial z_1}{\partial \gamma} \frac{\partial \gamma}{\partial k} < 0,$$

we have:

$$\begin{aligned} \frac{\partial z_1}{\partial m_1} &= \frac{\partial \gamma}{\partial m_1} x + \gamma \frac{\partial x}{\partial m_1} > 0, \\ \frac{\partial x}{\partial A} &= \left\{ \frac{\partial z_1}{\partial A} \gamma - z_1 \frac{\partial \gamma}{\partial A} \right\} \gamma^{-2} > 0, \\ \frac{\partial x}{\partial k} &= \left\{ \frac{\partial z_1}{\partial k} \gamma - z_1 \frac{\partial \gamma}{\partial k} \right\} \gamma^{-2} < 0. \end{aligned}$$

E4. Let us prove that $\partial f / \partial \beta > 0$, $\partial f / \partial (p/q) < 0$, $\partial f / \partial m_1 > 0$, $\partial f / \partial k < 0$.

The first sector output is $f = \Omega(\bar{x}) = \Omega(x - \mu(z_1/m_1)z_1)$, where \bar{x} is actual (*i.e.* net of transaction costs) production input. $\partial \Omega / \partial \bar{x} > 0$, and thereby the signs of the derivatives of f with respect to β , p/q , m_1 and k will coincide with the signs of the derivatives of \bar{x} with respect to β , p/q , m_1 and k .

$$\begin{aligned} \bar{x} &\equiv x - a(z_1/m_1)^\nu z_1 = \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{-1/(1-\alpha)}, \\ \frac{\partial \bar{x}}{\partial \gamma} &= (-1/(1-\alpha)) \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{-(2-\alpha)/(1-\alpha)} \frac{k}{A\alpha} \frac{\partial V(\gamma)}{\partial \gamma} > 0, \end{aligned}$$

and since \bar{x} depends on m_1 only through γ

$$\frac{\partial \bar{x}}{\partial m_1} = \frac{\partial \bar{x}}{\partial \gamma} \frac{\partial \gamma}{\partial m_1} > 0,$$

$$\frac{\partial \bar{x}}{\partial A} = -\frac{1}{1-\alpha} \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{-(2-\alpha)/(1-\alpha)} \frac{k}{\alpha} \left\{ \frac{\partial V(\gamma)}{\partial \gamma} \frac{\partial \gamma}{\partial A} \frac{1}{A} - \frac{V(\gamma)}{A^2} \right\},$$

$$\frac{\partial \bar{x}}{\partial A} > 0 \text{ if } \left| \left\{ \gamma \frac{\partial V(\gamma)}{\partial \gamma} : V(\gamma) \right\} \left\{ \frac{1}{\gamma} \frac{\partial \gamma}{\partial A} A \right\} \right| < 1,$$

i.e. the elasticity of $V(\gamma)$ with respect to γ and the elasticity of γ with respect to A in absolute terms are less than one.

$$\frac{\partial \bar{x}}{\partial k} = -\frac{1}{1-\alpha} \left\{ \frac{kV(\gamma)}{A\alpha} \right\}^{-(2-\alpha)/(1-\alpha)} \left\{ \frac{\partial V(\gamma)}{\partial \gamma} \frac{\partial \gamma}{\partial k} \frac{k}{A\alpha} + \frac{V(\gamma)}{A\alpha} \right\},$$

$$\frac{\partial \bar{x}}{\partial k} < 0 \text{ if } \left| \left\{ \gamma \frac{\partial V(\gamma)}{\partial \gamma} : V(\gamma) \right\} \left\{ \frac{1}{\gamma} \frac{\partial \gamma}{\partial k} k \right\} \right| < 1,$$

i.e. the elasticity of $V(\gamma)$ with respect to γ and the elasticity of γ with respect to k in absolute terms are less than one.

E5. Let us prove that $D = D(g, p/q, \varepsilon/q, \Theta, M_0/q, \gamma, \beta, k, cr_1, cr_2)$.

It is convenient to write the budget constraint in the form

$$\frac{p}{q}c + \frac{p}{q}m = (1-\theta) \left\{ \gamma \frac{p}{q}d \right\} + \frac{M_0}{q}.$$

Using the balance condition: $D = d = g + c(p/q, l/q)$.

It is obvious that $\partial d/\partial g > 0$, $\partial d/\partial \theta < 0$, $\partial d/\partial (M_0/q) > 0$.

The influence of γ , β , and k on demand is ambiguous due to the ambiguity of the variables' effect on the export function ξ . An increase in real credit limits to the first sector (cr_1) increases income through ξ and γ , thus raising demand. A relaxation of real credit limit to the second sector cr_2 decreases ξ , thus diminishing income and demand.

The sign of $\partial d/\partial (p/q)$ depends on the relative strength of the income and substitution effects:

$$\begin{aligned} \frac{\partial d}{\partial (p/q)} &= \frac{\partial c}{\partial (p/q)} + \frac{\partial c}{\partial (l/q)} \frac{\partial (l/q)}{\partial (p/q)} = \\ &= \frac{\partial c}{\partial (p/q)} + (1-\theta) \frac{\partial c}{\partial (l/q)} \left\{ \frac{\partial (\varepsilon q^0 \xi / q)}{\partial (p/q)} + \left[\gamma d + \frac{p}{q} \frac{\partial \gamma}{\partial (p/q)} d + \frac{p}{q} \gamma \frac{\partial d}{\partial (p/q)} \right] \right\}, \end{aligned}$$

$$\frac{\partial d}{\partial(p/q)} = \left\{ \frac{\partial c}{\partial(p/q)} + (1-\theta) \frac{\partial c}{\partial(l/q)} \left[\frac{\partial(\varepsilon q^0 \xi / q)}{\partial(p/q)} + \left(\gamma d + \frac{p}{q} \frac{\partial \gamma}{\partial(p/q)} d \right) \right] \right\} \times$$

$$\times \left\{ 1 - (1-\theta) \frac{p}{q} \gamma \frac{\partial c}{\partial(l/q)} \right\}^{-1},$$

where

$$\frac{\partial(\varepsilon q^0 \xi / q)}{\partial(p/q)} = \frac{\varepsilon q^0}{q} \frac{\partial \xi}{\partial(p/q)} < 0.$$

Provided that c is a normal good and assuming that the substitution effect is stronger than the income effect, $\partial D / \partial(p/q) < 0$.

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